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Application of Harmonic Analysis in the Preliminary Prediction of Air Temperature over Lagos and Abuja, Nigeria

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Abstract

Harmonic Analysis technique has been employed in predicting the hourly air temperature variations over Lagos and Abuja, Nigeria. The variations in hourly air temperatures over the two stations are periodic and thus have strong tendency of being repeated the next day, if all other atmospheric variables are constant. It was observed that the variation in hourly air temperature in the two stations is dominated by the first harmonic, thus it fluctuates by one cycle with a period of 24 hours. The maximum hourly air temperature occurred 2 hours on the average after the maximum solar irradiance has occurred in each station. The degree of hotness or coldness of the air at a later hour depends on that of the previous hours for each station.

Key words: *Harmonic analysis, air temperature, prediction, solar irradiance and Nigeria.*

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Introduction

The climate system adjusts when one or more of external factors change, for example, global average temperatures would be expected to increase with an increase in solar output. Climatic predictions are made using climate system models as the Atmosphere-Ocean general circulation models (AOGCMs). These are mathematical expressions of the thermodynamics; fluid motions; chemical reactions; and radiative transfer of the complete climate system that are as comprehensive as allowed by computational feasibility and scientific understanding of their formulation. The ultimate aim is to model as much as possible the climate system, especially the complex feedbacks among the various components. A number of models are in operation in various research institutes and universities Worldwide. Although the models are based on the same laws of physics, each has different ways of dealing with processes that cannot be represented explicitly by physical laws, such as formation of clouds and precipitation. Variations in these parameterizations lead to different regional projection of climate change, particularly for precipitation.

AOGCMs as well as many other climatic models cannot simulate all aspects of climate. The AOGCMs in particular have large uncertainties associated with clouds, it has difficulty portraying accurately precipitation patterns in mountainous regions and resolving important synoptic weather features (such as Mesoscale Convective Systems) that strongly influence precipitation patterns and amounts in many agricultural regions (Ebi and Means, 2002). Since the challenges of climate change persist globally, the need for climatic models and better statistical methods for analyzing climatic variables will continue to grow. This study in its own way of improving on existing methods of analyzing climatic variables, did not stop at the use of descriptive statistics (e.g. mean) in the analysis, but proceeds to developing a computer program for predicting the average hourly temperature over Lagos and Abuja, Nigeria.

Purpose of Study

The identification and implementation of methods that will effectively enhance the study of atmospheric variables has posed a challenge over time. Thus the aim of this work is to employ the use of harmonic analysis technique in fitting a periodic function or model to carry out a preliminary prediction of hour to hour air temperature over Lagos and Abuja, Nigeria.

Lagos is in the south western part of Nigeria and lies approximately on Longitude 2^0 42" and 3^0 22" East and between Latitude 6^0 22" and 6^0 42" North. It stretches over along the Guinea Coast of the Bight of Benin on the Atlantic Ocean. On the other hand, Abuja is in the northern part of Nigeria at Latitude 8.9 and Longitude 7.09. So the locations of these two cities make them liable to climatic variations due to ocean and desert effects respectively. Hence the work will help in providing useful information that will improve human and agricultural activities in these areas especially in this era of global erratic climatic change.

According to Weather fundamentals (2007), the amount of solar energy received by any region of the earth varies with time of day, with seasons, and with latitude. These differences in solar energy create temperature variations. These temperature variations create forces that drive the atmosphere in its endless motions.

Review of related works

In the past, some researchers have actually used harmonic analysis in studying temperature variations but none has been known to be carried out in Nigeria, especially with the Nigerian Environmental Climatic Observing Program (NECOP), real time data.

Lidija (2007) used harmonic analysis to investigate the seasonal cycle in temperature at five locations over the Mount Biokovo region (Croatia), Makarska, Opuzen, Sestanovac, Imotski, and Vigorac. The monthly averages of temperature as well as monthly means of minimum and maximum temperatures from 1961 to 1980 were subjected to harmonic analysis. The results were reported to have a good implication for botanical investigations.

Aisu and David (2000), used mean monthly average daily values of global irradiation for seven stations in Oman to develop harmonic models. Pertinent amplitudes and phase angles were obtained for each station. The results show the dominance of the first two harmonics in the southern stations, Salalah and Marmul and the dominance of only the first harmonic in the north, Buraimi and Seeb.

The mean annual variations of the air temperature over European and Mediterranean area have been studied using Harmonic Analysis. Basic data consist of the mean monthly values of air temperature from 1961-1990. It was found that the first 2 harmonic terms contribute, altogether, to the total variance of over 95% (Makrogiannis and Balafoutis, 2001).

Harmonic analysis along side with other statistical methods has also been applied to study seasonal variability of the air temperature at Mlynany using mean hourly, monthly and annual values of the air temperature during the period of 1962-2002. The results showed that the air temperature trend at Mlynany has an increasing tendency and that the mean annual air temperature increased by about 1.4 during the investigated period, i.e., approximately by 0.34^0 per decade.

Air-water temperature relationship in the Illinois River (Peoria) was studied using Harmonic analysis. It's application to daily mean air, water temperature records for this location, indicates that the first harmonic accounts for a major portion of the total variance in the records. It was discovered that water temperature residuals are well correlated with air temperature residuals. This result enabled the development of a mathematical model whose parametric values were used for predicting water temperatures from air temperature records and this was seen to be stable from year to year (Kethandaraman, 2007).

However in this work, the harmonic analysis technique is used in analyzing and predicting hourly air temperature data for a few days over Lagos and Abuja, Nigeria.

Source of data

The air temperature and solar irradiance data used for this work was obtained from Nigerian Environmental Climatic Observing Program (NECOP), NECOP is a new programme about three years old, designed to establish a network of meteorological and climatological observing stations spatially located across Nigeria. NECOP's main objectives among other things is to make real time data available for meteorological and climatological research which will serve as a warning tool for decision makers involved in emergency management, natural resources management, transportation and agriculture. The size of the NECOP real time data obtained in these stations is small. Hence Abuja and Lagos have 8 and 2 months data respectively. This does not allow for a long time climatic prediction of the area. Hence, this research serves as a preliminary investigation to the climatic prediction in these areas.

This work was carried out based on series equation adapted from Panofsky and Brier, 1960:

$$X_t = \bar{X} + \sum_{i=l}^{N/2} \left[A_i \sin\left(\frac{360}{P} it\right) + B_i \cos\left(\frac{360}{P} it\right) \right] \qquad (1)$$

Where N is the number of observations, X1 = the time series, X = arithmetic mean, P= period of observation.

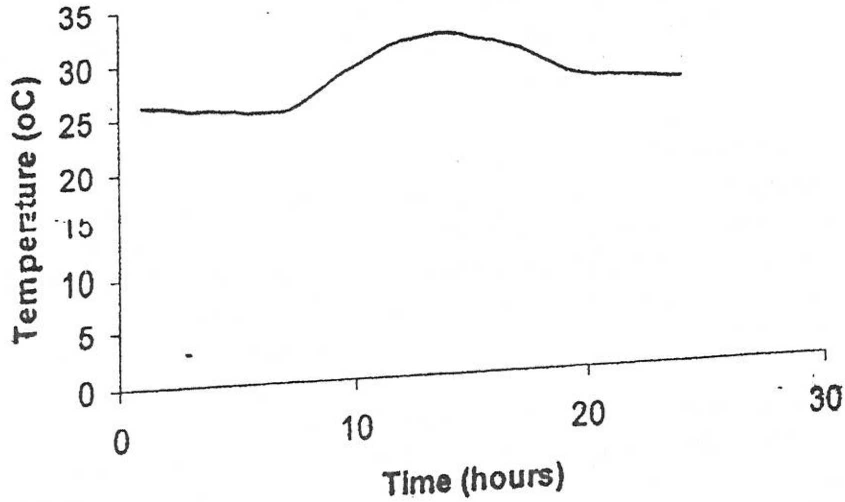
$$A = \frac{2}{N} \sum_{i=l}^{N/2} \left[X \sin\left(\frac{360}{P} it\right) \right],\ B = \frac{2}{N} \sum_{i=l}^{N/2} \left[X \cos \frac{360}{P} it \right] \qquad (2)$$

A and B are coefficients
X = Observed value, t = time and i = number of harmonies.
In other words the time series equals the mean plus the sum of all N/2 harmonies.

The first harmonic (or fundamental) has a period equal to the total period studied. The second harmonic has a period equal to half the fundamental period, the third harmonic a period of one = third of the fundamental and so on. It is not always required to determine all the N/2 harmonies, usually the first two, or at most three, harmonies describe the variation of the periodic sufficiently well. The equation (1) can be re-written as:

$$X_t = \bar{X} + \sum_{i=l}^{N/2} C_i \cos\left[\frac{360}{P} i\,(t - t_i)\right] \qquad (3)$$

Where $C_i = \sqrt{A^2 + B^2}$ is the amplitude of the i[th] harmonic and



The result of the run of the harmonic analysis program on the hourly average air temperature data is summarised in Table 1.

Table 1: Result extracts from the run of harmonic analysis program on the hourly average air of temperature of Lagos

Harmonics	Since Coefficients (A_i)			Cosine Coefficients (B_i)			Amplitude	Time Harmonic is Maximum (h)	Percentage contribution
	A1	A2	A3	B1	B2	B3			
1st	-1.7720	0	0	-2.2585	0	0	2.87	14.54	86.72
2nd	0	0.6593	0	0	0.8699	0	1.09	13.24	12.54
3rd	0	0	0.0746	0	0	0-.1017	0.13	13.07	0.17

Using the average daily air temperature of 27.810C, the period of 24 hours and the sine and cosine coefficients of Table 1, the fitted periodic function X1 for the hourly average temperature of Lagos station is obtained using extracts from equation 1 as:

$$X_i = 27.81 + \sum_{i=1}^{3}[A_i \sin(15it) + B_i \cos(15it)] \quad (4)$$

Where A_1 and B_1 are the coefficients of the sine and cosine respectively and the I's are integers ranging from 1 to 3 as given in Table 1.

Equation 4 was employed in the harmonic analysis program so as to make a six days forecast of hourly average air temperature measurements for Lagos station. The result of this forecast is displayed with the corresponding actual and estimates of hourly average air temperature measurements in Table 2.

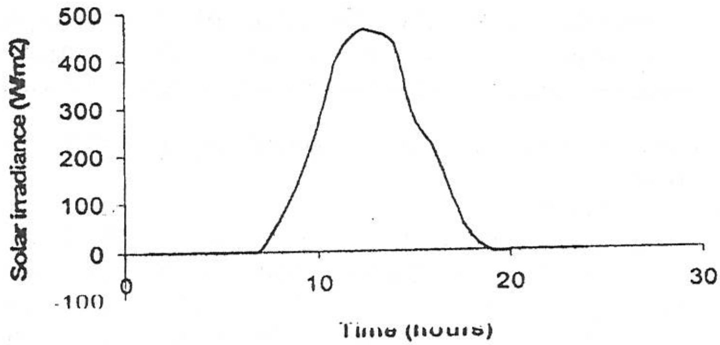
Table 2: A display of six day forecasts of hourly average air temperature for Lagos station against corresponding actual and model estimate values

Time (hours)	Actual values	Model estimate	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
1	26.12	26.23	26.23	26.23	26.23	26.23	26.23	26.23
2	25.91	26.05	26.05	26.05	26.05	26.04	26.05	26.05
3	25.68	25.74	25.74	25.74	25.74	25.74	25.74	25.74
4	25.53	25.38	25.38	25.38	25.38	25.38	25.38	25.38
5	25.32	25.11	25.11	25.11	25.11	25.11	25.11	25.11
6	25.19	25.09	25.09	25.09	25.93	25.09	25.09	25.09
7	25.13	25.48	25.48	25.48	25.48	25.48	25.48	25.48
8	26.11	26.30	26.30	26.30	26.30	26.30	26.30	26.30
9	27.67	27.48	27.48	27.48	27.48	27.48	27.48	27.48
10	28.96	28.89	28.82	28.82	28.82	28.82	28.83	28.83
11	30.15	30.08	30.08	30.08	30.09	30.09	30.09	30.09
12	30.97	31.04	31.04	31.04	31.04	31.04	31.04	31.05
13	31.46	31.55	31.55	31.55	31.55	31.55	31.55	31.55
14	31.54	31.58	31.58	31.58	31.58	31.58	31.58	31.58
15	31.09	31.19	31.19	31.19	31.19	31.19	31.19	31.19
16	30.61	30.51	30.51	30.51	30.50	30.50	30.50	30.50
17	29.90	29.66	29.66	29.66	29.66	29.66	29.66	29.66
18	28.82	28.78	28.78	28.78	28.78	28.78	28.78	28.78
19	27.69	27.98	27.98	27.98	27.97	27.97	27.97	27.97
20	27.19	27.31	27.31	27.31	27.31	27.31	27.31	27.31
21	26.91	26.82	26.82	26.82	26.82	26.82	26.82	26.82
22	26.66	26.52	26.53	26.53	26.53	26.53	26.53	26.53
23	26.49	26.38	26.38	26.38	26.38	26.38	26.38	26.38
24	26.29	26.32	26.32	26.32	26.32	26.32	26.32	26.32

Daily average	27.81	27.81
Standard deviation	2.18	2.17

Analyses and results for Abuja Station

The same process was used to carry out the analysis for Abuja and the results for the time series plots of air temperature and solar irradiance measurements are given in Figures 4 and 5 respectively.



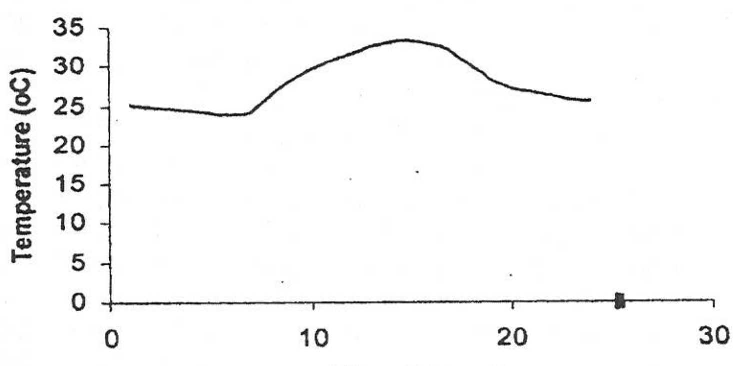


Figure 5: Time series plot of hourly average solar irradiance for Abuja Station. The fitted periodic function X_1 for the hourly average temperature of Abuja Station is given as:

$$X_t = 27.89 + \sum_{i-i}^{i}[A_i \sin(15it) + B_i \cos(15it)] \qquad (5)$$

Where A1 and B1 are the coefficients of the sine and cosine respectively and the I's are integers ranging from 1 to 3 as given in Table 3.

Table 3: Result extracts from the run of harmonic analysis program on the hourly average air of temperature of Abuja Station

Harmonics	Since Coefficients (A_i)			Cosine Coefficients (B_i)			Amplitude	Time Harmonic is Maximum (h)	Percentage contribution
	A1	A2	A3	B1	B2	B3			
1st	-2.8	0	0	-3.34	0	0	4.37	15.07	91.69
2nd	0	0.89	0	0	0.79	0	1.19	14.02	6.83
3rd	0	0	0.30	0	0	0.22	0.37	13.17	0.65

The result of six days forecast of hourly average air temperature measurements for Abuja is displayed with the corresponding actual and model estimates of hourly average air temperature measurements in Table 4.

Table 4: A Time distribution of Maximum air temperature and Maximum Solar irradiance across Stations

Station	Maximum Air Temperature (°C)	Time (hours)	Maximum Solar Irradiance (W/m^2)	Time (hours)
Lagos	31.53841	14h (2P.M)	457.2712	12h (Noon)
Abuja	33.17894	15h (3 P.M)	511.1827	12h (Noon)

Discussions

Observations show that the equation (1) – (6) exhibit a good fit to the hourly average temperature of Lagos and Abuja, as thy produced very close estimates of the actual hourly average air temperatures, yield the same mean (average daily air temperature) as that of the actual data and very close standard deviation of the actual data and that of the model estimates as shown in Table 2 and 4 respectively. It can also be observed from Tables 1 and 3 that the first harmonic dominates the periodic components in the hourly average air temperature of all the Stations since it has the highest contribution of 86.72% and 91.69% for Lagos and Abuja stations respectively, showing that the hourly air temperature of each station fluctuates by one cycle. This implies that the contributions of the second and third harmonies for each Station is negligible thus, the information about them is discarded.

The six days forecasts of hourly average temperatures for each Station as shown in Tables 3, 2 and 4 respectively, show a strong indication that the hourly air temperatures across the two stations have the tendency of being repeated in every twenty four hours provided all other intervening atmospheric variables are kept constant. The results in Tables 1 and 3, show that the first harmonic is maximum at time; 14.54^{th} and 15.07^{th} for Lagos and Abuja respectively, which give close estimate of the time at which the actual value of the maximum hourly average air temperature occurred for each station as shown in Table 5. This result further validates the fitted periodic function to the data for each Station.

Figure 2 and 5 show that the solar irradiance for each station is low in the morning hours, highest in the afternoon and again low towards the evening hours. For the Lagos station, solar heating starts increasing from 7^{th} (7a.m), peak at 12^{th} (12 noon), and reduces from 13h (1p.m) while in Abuja Station, solar heating also starts increasing from 6h (6a.m), peak at 12h (12 noon), and reduces from 13h (1p.m). Figures 1 and 4 show the fluctuation in average hourly air temperatures for each station. For the Lagos the air temperature starts increasing at 8h (8a.m), peaks at 14h (2p.m) and reduces from 15h (3 p.m). The air temperature of the Abuja station starts increasing at 7h (7a.m), peaks at time 15h (3p.m) reduces from 14h (2p.m).

It can be observed from Table 5 that the maximum air temperatures is not the same across the two stations and neither did it occur at the time for which the solar irradiance is maximum for each station. The maximum average hourly temperature occurs 2hrs after the maximum average hourly solar heating had occurred in Lagos, while after 3hrs in Abuja. This could be attributed to the fact that the stations differ in the nature of geophysical features they are endowed with. These physical features such as water bodies, hills and mountains differ in their specific heat capacities.

The rate of radiation loss at the different locations could be explained by the Stefan-Boltzman law, which states that: "the total energy radiated per unit surface area of a black body in unit time is directly proportional to the fourth power of its thermodynamic or surface temperature". Thus, since these surfaces differ in their surface temperatures, their respective radiation loses cannot be equal to the solar irradiance at the same time, affirming the reason why maximum air temperatures did not occur at the same time of the day across all stations and neither did it occur at the time for which the solar irradiance is maximum for each station. This could also be explained by a phenomenon known as solar and terrestrial balance which asserts that the maximum air temperature occurs at the time when the solar heating and the energy lost by these surfaces (terrestrial radiation) are equal. In addition, amount of solar energy received by any region varies with the time of the day, season and latitude and these difference in solar energy create temperature variations (Weather fundamentals, 2007).

Conclusion

From the results of the analyses made for each station, the following conclusion can be drawn:

(i) The variations in hourly air temperatures across Lagos and Abuja, Nigeria are periodic and thus have strong tendency of being repeated the next day, if all other atmospheric variables are constant.

(ii) The hourly air temperatures and solar irradiance vary across each station, and that their respective maximum air temperatures never occurred at the time when the solar irradiance is maximum.

(iii) The maximum hourly air temperature occurs 2hrs on the average after the maximum solar irradiance has occurred in each station.

(iv) The degree of hotness or coldness of the air at a later hour depends on that of the previous hours for each station.

(v) The Nature of substances (hills, water bodies, mountains, etc) could have effects in variations in the average hourly temperature across stations.

(vi) The harmonic equations exhibit a good fit to the hourly average temperature of Lagos and Abuja and hence can be applied for hourly temperature prediction irrespective of the length of the data.

Acknowledgement

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